AP Calculus BC — Graphical Analysis

2010 #5

a) $g(3) = 5 + \int_0^3 g'(t)\,dt$ $\qquad g(-2) = 5 - \frac{1}{4}\pi(2)^2$

$= 5 + \frac{1}{4}(\pi)(2)^2 + \frac{1}{2}(1)(3)$

b) g'' changes signs @ $x=0$, $x=2$, $x=3$ $\Rightarrow g$ has a P.o.I.

c) $h(x) = g(x) - \frac{1}{2}x^2$

$h'(x) = g'(x) - x = 0$

$g'(x) = x$

on $[0,2]$

$\sqrt{4-x^2} = x$

$4 - x^2 = x^2$

$4 = 2x^2$

$x = \sqrt{2}$

max, $h'(x)$ Δs from + to −

on $[2,3]$

$g'(x) = x$ @ $x = 3$

neither, $h'(x)$ doesn't change signs.

$h'(x)$: $+$ | $\sqrt{2}$ | $-$ | 3 | $-$

2002 (Form B) #4 $\qquad g(x) = 5 + \int_6^x f(t)\,dt$

a) $g(6) = 5$ $\qquad g'(x) = f(x)$ $\qquad g''(x) = f'(x)$

$g'(6) = f(6) = 3$ $\qquad g''(6) = f'(6) = 0$

b) g is decreasing on $[-3, 0)$ & $(12, 15]$ b/c $g'(x) = f(x) < 0$.

c) g is concave down on $(6, 15)$ b/c $g''(x) = f'(x) < 0$

d) $\int_{-3}^{15} f(t)\,dt \approx \frac{3}{2}\left[f(-3) + 2f(0) + 2f(3) + 2f(6) + 2f(9) + 2f(12) + f(15)\right]$

$= \frac{3}{2}\left[-1 + 0 + 2(1) + 2(3) + 2(1) + 0 - 1\right]$

2013 BC #3 $\qquad g(x) = \int_{-3}^{x} f(t)\,dt \rightarrow g'(x) = f(x) \rightarrow g''(x) = f'(x)$

a) $g(3) = \int_{-3}^{3} f(t)\,dt = \frac{1}{2}(5)(4) - \frac{1}{2}(1)(2) = 9$

b) $g'(x) > 0$ on $(-5, 2)$; $g''(x) < 0$ on $(-5, -3)$ & $(0, 4)$

g is inc & concave down on $(-5, -3)$ & $(0, 2)$

c) $h(x) = \frac{g(x)}{5x}$

$h'(x) = \frac{5x\,g'(x) - 5g(x)}{25x^2}$

$h'(3) = \frac{15(-2) - 45}{25(9)}$

d) $p(x) = f(x^2 - x)$

$p'(x) = (2x-1) \cdot f'(x^2 - x)$

$p'(-1) = -3 \cdot f'(2)$

$p'(-1) = -3(-2) = 6$

2014 BC #4 $g(x) = 2x + \int_0^x f(t)\,dt$

$g'(x) = 2 + f(x)$

$g''(x) = f'(x)$

a) $g(-3) = -6 + \int_0^{-3} f(t)\,dt$

$= -6 - \frac{1}{4}\pi(3)^2$

b) Abs max @ endpoints or critical points

$\underline{x=-4}$

$g(-4) = -8 + \int_0^{-4} f(t)\,dt$

$= -8 - \frac{9\pi}{4} + \frac{\pi}{4}$

$= -8 - 2\pi$

$g'(x) = 0$

$2 + f(x) = 0$

$f(x) = -2$ @ $x = 2.5$

$g(2.5) = 5 + \int_0^{2.5} f(t)\,dt$

$= 5 + \frac{1}{2}(\frac{3}{2})(3) - \frac{1}{2}(1)(2)$

ABS MAX

$\underline{x=3}$

$g(3) < g(2.5)$

c) $g(x)$ has a P.o.I @ $x = 0$ b/c g'' changes signs

d) $\frac{f(3)-f(-4)}{3+4} = \frac{-3+1}{7} = -\frac{2}{7}$ $f(x)$ is not diff. on $(-4, 3)$ so MVT is not guaranteed.

2016 BC #3 $g(x) = \int_2^x f(t)\,dt$ $g'(x) = f(x)$ $g''(x) = f'(x)$

a) g has neither max nor min @ $x = 10$ b/c $g'(x)$ doesnot Δ signs

b) g has a PoI @ $x=4$ b/c g'' changes signs

c) $g' = 0$ @ $x = -2$, $x = 6$ ($x=2$ & $x=10 \rightarrow$ no signs change)

$\underline{x=-4}$

$g(-4) = -\frac{1}{2}(4)(4) + \frac{1}{2}(2)(4)$

$= -8 + 4$

$= -4$

$\underline{x=-2}$

$g(-2) = -\frac{1}{2}(4)(4)$

$= -8$

Abs Min

$\underline{x=6}$

$g(6) = \frac{1}{2}(4)(4)$

$= 8$

Abs. Max

$\underline{x=12}$

$g(12) = -4$

d) $(-4, 2)$ & $(10, 12)$