

AP Calculus BC

Graphical Analysis

2010 #5

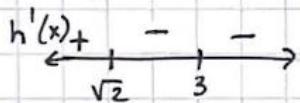
a) $g(3) = 5 + \int_0^3 g'(t) dt$ $g(-z) = 5 - \frac{1}{4}\pi(z)^2$
 $= 5 + \frac{1}{4}(\pi/2)^2 + \frac{1}{2}(1)(3)$

b) g'' changes signs @ $x=0, x=2, x=3 \Rightarrow g$ has a P.o.I.

c) $h(x) = g(x) - \frac{1}{2}x^2$ on $[0, 2]$ on $[2, 3]$

$$h'(x) = g'(x) - x = 0$$

$$g'(x) = x$$



$$\begin{aligned} \sqrt{4-x^2} &= x \\ 4-x^2 &= x^2 \\ 4 &= 2x^2 \\ x &= \sqrt{2} \end{aligned}$$

$$g'(x) = x @ x = 3$$

neither, $h'(x)$
doesn't change signs.

max, $h'(x)$ at
from + to -

2002 (Form B) #4 $g(x) = 5 + \int_6^x f(t) dt$

a) $g(6) = 5$ $g'(x) = f(x)$ $g''(x) = f'(x)$
 $g'(6) = f(6) = 3$ $g''(6) = f'(6) = 0$

b) g is decreasing on $[-3, 0] \cup (12, 15]$ b/c $g'(x) = f(x) < 0$.

c) g is concave down on $(6, 15)$ b/c $g''(x) = f'(x) < 0$

d) $\int_{-3}^{15} f(t) dt \approx \frac{3}{2} [f(-3) + 2f(0) + 2f(3) + 2f(6) + 2f(9) + 2f(12) + f(15)]$
 $= \frac{3}{2} [-1 + 0 + 2(1) + 2(3) + 2(1) + 0 - 1]$

2013 BC #3 $g(x) = \int_{-3}^x f(t) dt \rightarrow g'(x) = f(x) \rightarrow g''(x) = f'(x)$

a) $g(3) = \int_{-3}^3 f(t) dt = \frac{1}{2}(5)(4) - \frac{1}{2}(1)(2) = 9$

b) $g'(x) > 0$ on $(-5, 2)$; $g''(x) < 0$ on $(-5, -3) \cup (0, 4)$
 g is inc & concave down on $(-5, -3) \cup (0, 2)$

c) $h(x) = \frac{g(x)}{5x}$

$$h'(x) = \frac{5xg'(x) - 5g(x)}{25x^2}$$

$$h'(3) = \frac{15(-2) - 45}{25(9)}$$

d) $P(x) = f(x^2 - x)$

$$P'(x) = (2x-1) \cdot f'(x^2 - x)$$

$$P'(-1) = -3 \cdot f'(-2)$$

$$P'(-1) = -3(-2) = 6$$

2014 BC #4

$$g(x) = 2x + \int_0^x f(t) dt$$

$$\begin{aligned} g'(x) &= 2 + f(x) \\ g''(x) &= f'(x) \end{aligned}$$

$$\begin{aligned} a) g(-3) &= -6 + \int_0^{-3} f(t) dt \\ &= -6 - \frac{1}{4}\pi(3)^2 \end{aligned}$$

b) Abs max @ endpoints or critical points

$$\begin{aligned} x &= -4 \\ g(-4) &= -8 + \int_0^{-4} f(t) dt \\ &= -8 - \frac{9\pi}{4} + \frac{\pi}{4} \\ &= -8 - 2\pi \end{aligned}$$

$$\begin{aligned} g'(x) &= 0 \\ 2 + f(x) &= 0 \end{aligned}$$

$$f(x) = -2 @ x = 2.5$$

$$\begin{aligned} g(2.5) &= 5 + \int_0^{2.5} f(t) dt \\ &= 5 + \frac{1}{2}(\frac{3}{2})(3) - \frac{1}{2}(1)(2) \end{aligned}$$

$$\begin{aligned} x &= 3 \\ g(3) &< g(2.5) \end{aligned}$$

Abs Max

c) $g(x)$ has a P.o.I @ $x=0$ b/c g'' changes signs

d) $\frac{f(3)-f(-4)}{3+4} = \frac{-3+1}{7} = -\frac{2}{7}$ $f(x)$ is not diff. on $(-4, 3)$ so MVT is not guaranteed.

2016 BC #3

$$g(x) = \int_2^x f(t) dt \quad g'(x) = f(x) \quad g''(x) = f'(x)$$

a) g has neither max nor min @ $x=10$ b/c $g'(x)$ does not change signs

b) g has a P.o.I @ $x=4$ b/c g'' changes signs

c) $g'=0 @ x=-2, x=6$ ($x=2$ & $x=10 \rightarrow$ no signs change)

$$\begin{array}{lll} \underline{x=-4} & \underline{x=-2} & \underline{x=6} \\ g(-4) = -\frac{1}{2}(4)(4) + \frac{1}{2}(2)(4) & g(-2) = -\frac{1}{2}(4)(4) & g(6) = \frac{1}{2}(4)(4) \\ = -8 + 4 & = -8 & = 8 \\ = -4 & \underline{\text{Abs Min}} & \underline{\text{Abs Max}} \end{array}$$

d) $(-4, 2) \notin (10, 12)$